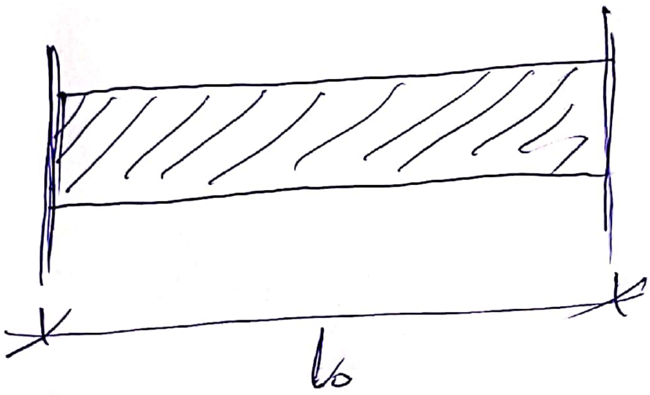
TEMPERATURE STRESSES

With rise or fall in temperatures, materials expand or contract. Resistance, in any form, to this expansion or contraction will set up stresses in the material

TEMPERATURE STRESSES FOR SINGLE BAR

The bar shown below is restrained at both ends, and it is undergoing variations in temperature



Let

l_0 = original length of the bar

t = rise in temperature

α = coefficient of expansion for the material of the rod

Now if after temperature rise $t°C$, the length of the bar is l_t

The change in length δl

$$\delta = l_o \alpha t \qquad (1)$$

So that the total length after temperature change is

$$l_t = l_o + l_o \alpha t$$
$$= l_o(1 + \alpha t) \qquad (2)$$

Also $\quad l_t - l_o = l_o \alpha t \qquad (3)$

But the strain

$$\epsilon = \frac{\text{change in length}}{\text{original length}} = \frac{l_o \alpha t}{l_o}$$

$$= \alpha t \qquad (4)$$

Thus the stress (σ) due to the strain, from Hooke's Law

$$\epsilon = \frac{\sigma}{E}$$

$$\sigma = \epsilon E$$
$$= (\alpha t) E \quad (\text{from eqn } 4)$$
$$= \alpha t E \qquad (5)$$

Thus eqns 4 & 5 are the strain and stress induced by temperature variations in a single Bar

Example

If a rod of length 2m at the temperature of 10°C has its temperature raised to 80°C, find the expansion in the rod. Find also the stress in the rod if expansion is ~~prevented~~ prevented. Take E = 100 & $\alpha = 0.000012/°C$

Solution

$l_0 = 2m = 2000$

$t = 80 - 10 = 70°C$

$\alpha = 0.000012/°C$

l_t = length at 80°c

$l_t = l_0(1 + \alpha t)$

change in length δl

$\delta l = l_t - l_0 = l_0(1+\alpha t) - l_0$

$= l_0 + \alpha t l_0 - l_0$

$= \alpha t l_0$

$= 0.000012 \times 70 \times 2000$

$= 1.68\,mm$

If expansion is prevented, strain is induced, and it is

$\epsilon = \frac{\delta l}{l_0}$ $\left\{\frac{\text{change in length}}{\text{original length}}\right\}$ $= \frac{1.68}{2000} = 0.00084$

Thus the stress caused is, knowing that $\epsilon = \frac{\sigma}{E}$

$\sigma = \epsilon E = 0.00084 \times 100 \times 10^9 = 84 \times 10^6\,N/m^2$